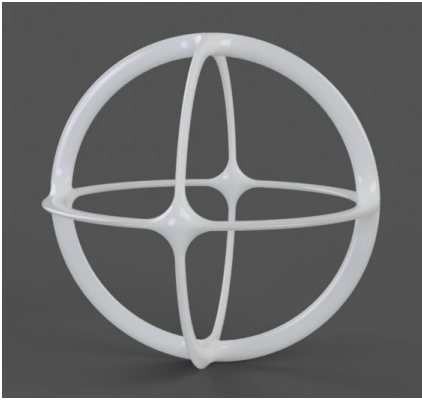


## Team 4



**Figure 1:** Proposed design of 3 interlocking tori with elliptical cross-sections. Joints are reinforced.

### Background

Our goal was to create a part to be 3D printed in plastic resin that will take 30 seconds to fall in a pipe with 0.25' of oil and 6' of water. The terminal velocity should be 0.0633 m/s in order for the object to have a descent time of 30 seconds, under the assumption that it reaches this velocity quickly. The density of the plastic resin ( $1180 \pm 20 \frac{\text{kg}}{\text{m}^3}$ ) is close to the density of water ( $1000 \frac{\text{kg}}{\text{m}^3}$ ).

### Design Space

Our intuition for this problem is that we would need to minimize terminal velocity by increasing drag. Some initial calculations for a sphere and streamline bodies confirmed this. For example, for a sphere to reach terminal velocity such that it would take 30 seconds to fall, it would have a diameter of 1 mm. We further tried a streamline body, with a drag coefficient of 0.04. This led to a 1.55 m/s velocity, which was much too fast. From this we verified that we needed to increase drag.

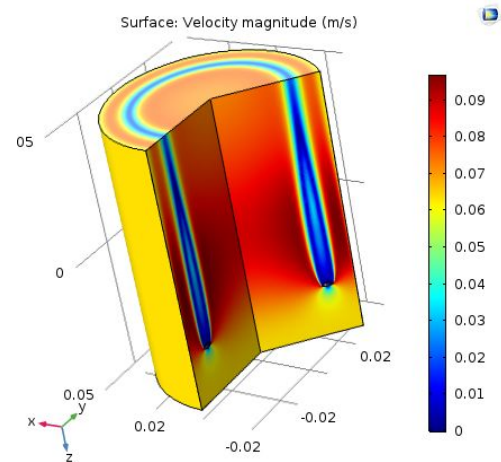
We initially tried designs that would generate angular velocity, or create significant drag. However these were geometrically complex, making it hard to simulate them computationally, or estimate analytically. We were also limited by not being able to do physical tests. Inspired by the streamline body which was too fast, and the sphere which was impractically small, we decided to increase drag by choosing a shape with a large surface area to volume ratio.

We settled on an elliptical torus, which allows us to maximize drag on the surface area, but minimize volume. However, one issue with this design is that the

orientation of the torus normal to gravity is unstable. We therefore decided to interlock 3 tori, each with the same elliptical cross-section. We predict that this will have an equilibrium position rotated 45 degrees about both the x and y axis (relative to the orientation shown in Figure 1).

### Calculations

We calculated that for the regime of this experiment, terminal velocity would be reached within one second. Given that drag force  $D = \frac{1}{2}C_D \rho_F U^2 A$  equals the difference between weight  $W = mg$  and buoyancy  $B = \gamma_F V$  at steady state, the equation for the drag coefficient would be represented as follows:  $C_D = [2m_O(g\rho_O - \gamma_F)] / (\rho_O \rho_F U_O^2 A_O)$ , where  $A_O$  is the projected cross-sectional area.



**Figure 2:** COMSOL simulation of an elliptical torus in a 3' pipe, illustrating the wake. Calculated  $C_D = 1.75$ .

To estimate the drag coefficient, we simulated a single elliptical torus in COMSOL. Given a torus with major radius 28 cm, and an ellipse with major axis 4 cm and minor axis 1.33 cm, we computed a  $C_D$  of 1.75. For our object, this results in a drag force of 0.0025 N, and given its volume, a weight of 0.0019 N. This is for a single torus, whereas our design uses three interlocking tori. We assume that for the tori *not* normal to gravity, they will have a smaller drag coefficient, but the same weight contribution. We predict that this will compensate for the higher drag force in the single torus case.